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312. Proposed by J. A. CAPARO, C. E., Notre Dame University, Notre Dame, Ind.

Two roots of the cubic  $x^3 - px^2 + qx - c = 0$  are equal. Find their value in terms of  $p$ ,  $q$ , and  $c$ .

I. Solution by GEORGE W. HARTWELL, University of Kansas, Lawrence, Kas.; V. M. SPUNAR, M. and E. E., East Pittsburgh, Pa.; and G. I. HOPKINS, Instructor in Mathematics and Astronomy, Manchester High School, N. H.

Let  $m$ ,  $m$ ,  $n$  be the three roots. Then

$$2m+n=p \dots (1); \quad 2mn+m^2=q \dots (2); \quad m^2n=c \dots (3).$$

Solving (1) and (2) for  $m$  and  $n$  we have,

$$m = \frac{p \pm \sqrt{(p^2 - 3q)}}{3}, \quad n = \frac{p \mp 2\sqrt{(p^2 - 3q)}}{3}.$$

Substituting these values in (3),

$$(2p^2 - 6q) \left( \frac{p \pm \sqrt{(p^2 - 3q)}}{3} \right) = pq - 9c \dots (4). \text{ But } \frac{p \pm \sqrt{(p^2 - 3q)}}{3} = m.$$

Therefore, from (4),  $m = \frac{pq - 9c}{2p^2 - 6q}$ .

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; S. LEFSCHETZ, Wilkinsburg, Pa.; and the PROPOSER.

Let  $f(x) = x^3 - px^2 + qx - c = 0$ ,  $f'(x) = 3x^2 - 2px + q = 0$ .

If  $f(x)$  has two equal roots,  $f'(x)$  contains one, and hence the greatest common divisor of  $f(x)$  and  $f'(x)$  gives one of the equal roots. Now if

$$(pq - 9c)(27c + 4p^3 - 15pq) = 4q(3q - p^2)^2, \text{ or}$$

$$18cpq + p^2q^2 - 4cp^3 - 4q^3 - 27c^2 = 0,$$

then  $2(3q - p^2)x + pq - 9c$  is the greatest common divisor of  $f(x)$  and  $f'(x)$ .

Therefore,  $x = (9c - pq) / [2(3q - p^2)]$  is one of the equal roots.

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## GEOMETRY.

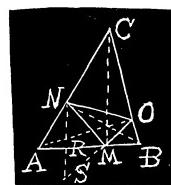
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339. Proposed by G. E. BROCKWAY, Boston, Mass.

Of all triangles that can be inscribed in a given triangle, that formed by joining the feet of the altitudes has the minimum perimeter. Prove by means of the straight line and circle.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Assuming the points  $N$  and  $O$  in the sides  $AC$  and  $BC$ ,  $NM+MO$  is a minimum, if  $\angle NMA = \angle OMB$ ; for letting fall the perpendicular  $NR$  and extending it to  $S$  by its own length,  $OMS$  becomes a straight line. It follows from this that in the case of  $MNO$  being a triangle of minimum perimeter,  $\angle NMA = \angle BMO = \alpha$ ,  $\angle ANM = \angle CNO = \beta$ ,  $\angle MOB = \angle NOC = \gamma$ .



Now,  $\alpha + \beta = 180^\circ - A$ ,  $\alpha + \gamma = 180^\circ - B$ ,  $\beta + \gamma = 180^\circ - C$ .  
 $\therefore \alpha + \beta + \gamma = \frac{1}{2}(540^\circ - 180^\circ) = 180^\circ$ .  $\therefore \alpha = C$ ,  $\beta = B$ ,  $\gamma = A$ . Consequently, the triangle  $MNO$  is the pedal triangle.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

When we wish to find a point  $P$  on a given line so that the sum of the distances  $PR+PS$  to two given points, is a minimum, it is easy to show that  $PS$  and  $PR$  must make equal angles with the given line. Let  $M, L, K$  be the feet of the altitudes;  $M$  on  $AB$ ,  $L$  on  $AC$ ,  $K$  on  $BC$ . Then if  $M, L$  are fixed,  $K$  is the point on  $BC$  such that  $KL+KM$  is a minimum, since  $KL, KM$  make equal angles with  $BC$ . Similarly, for  $M, K$  and  $L, K$  fixed in turn, respectively.  $LM+LK$  is a minimum and  $ML+MK$  is a minimum for the reason cited above.

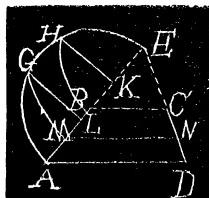
This seems the easiest and simplest proof.

340. Proposed by J. H. MEYERS, S. J., Sacred Heart College, Augusta, Ga.

Given trapezoid  $ABCD$ . Prolong  $AB$  and  $CD$ , the non-parallel sides, to meet in  $E$ . On  $AE$  as diameter construct semi-circle  $AHGE$ . With  $BE$  as radius construct arc  $BG$ . Draw  $GK$  perpendicular to  $AE$ . Bisect  $AH$  at  $L$ . Erect  $KH$  perpendicular to  $AE$ . Construct arc  $HM$  with  $HE$  as radius. Draw  $MN$  perpendicular to  $DC$ . Prove that  $MN$  bisects the trapezoid  $ABCD$ , angles  $ADC$  and  $BCD$  being right angles.\*

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

We may generalize this theorem by drawing the line  $MN$  parallel to  $AD$ , instead of perpendicular to  $DC$ .



$$\triangle ADE : \triangle EMN = AE^2 : EM^2 = AE^2 : EH^2 = AE^2 : AE \times EL = AE : EL.$$

$$\therefore \triangle ADE - \triangle EMN : \triangle EMN = AE - EL, \text{ or}$$

$$AMND : \triangle EMN = AL : EL \dots (\text{I}).$$

$$\triangle EMN : \triangle EBC = ME^2 : BE^2 = KE^2 : GE^2 = AE \times EL : AE \times EK = EL : EK.$$

$$\therefore \triangle EMN - \triangle EBC : \triangle EMN = EL - EK : EL, \text{ or}$$

$$MBCN : \triangle EMN = LK : EL \dots (\text{II}).$$

Comparing (I) and (II),  $AMND : MBCN = AL : LK = 1 : 1$ .

\*The reading of this problem has been slightly changed to correspond to the figure. ED. F.